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EFFECT OF BOUNDARY-LAYER INJECTION ON THE DRAG OF AN AXISYMMETRIC BODY IN A HYPERSONIC IMPERFECT-GAS FLOW

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The article examines a similarity problem of hypersonic flow of an imperfect gas about an axisymmetric body in the presence of boundary-layer injection.

We will study the basic principles underlying the effect of gas injection over a solid surface into a boundary layer on the drag of the axisymmetric body in a hypersonic gas flow by examining flow about a body with its generatrix described by the power relation $r_w(x) = ax^{3/4}$. It was shown in [1] that, in this case, the problem of the viscous interaction of the nonpermeable surface of the body with the imperfect heat-conveying gas has the property of similitude, and the gasdynamic parameters in the boundary layer depend on a single variable

$$\eta = \frac{\sqrt{\mathrm{Re}} \int\limits_{r_w}^{r} \rho r dr}{2\mathrm{M}_\infty \sqrt{\xi}}$$

These parameters are determined from a system of ordinary differential equations. When gas is injected over the surface of the body into the boundary layer, the similarity property of the problem is preserved if the distribution of the mass injection rate along the generatrix of the body is determined by the relation

$$(\rho v r)_w = a^4 \frac{\chi}{(\mathrm{M}_\infty a)^2} \sqrt{1 + kJ(\infty)} \frac{3\sqrt{c}}{4} \alpha. \tag{1}$$

In accordance with [1], the following notation is adopted:

$$\chi = \frac{\mathrm{M}_\infty^3}{\sqrt{\mathrm{Re}}}; \; \xi = \frac{1}{2} \int\limits_0^x p_\delta r_w^2 dx; \;\; \eta = \frac{\sqrt{\mathrm{Re}}\, u_\delta}{2\mathrm{M}_\infty \sqrt{\xi}} \int\limits_{r_w}^{r} \rho r dr; \; p_\delta = c\left(\frac{dr^*}{dx}\right)^2.$$

If a gas different from the gas of the hypersonic flow is injected into the boundary layer, then, besides (1), we need to assume that the viscosity coefficient of these gases under these conditions is a power function of temperature and that the exponent is the same for each gas. Then the differential equations to which the problem is reduced take the form

$$\frac{\partial}{\partial \eta}\left(YN \frac{\partial u}{\partial \eta}\right) + f \frac{\partial u}{\partial \eta} = 2m(g - u^2),$$

$$\frac{\partial}{\partial \eta}\left(\frac{YN}{\mathrm{Pr}} \frac{\partial g}{\partial \eta}\right) + f \frac{\partial g}{\partial \eta} = -\frac{\partial}{\partial \eta}\left(YN\left(1 - \frac{1}{\mathrm{Pr}}\right)\frac{\partial u^2}{\partial \eta}\right) - \frac{\partial}{\partial \eta}\left(\frac{1}{\mathrm{Sm}}\left(1 - \frac{1}{\mathrm{Le}}\right)\sum_i \frac{h_i - h_i^0}{H_\delta} \frac{\partial c_i}{\partial \eta}\right), \tag{2}$$

$$\frac{\partial}{\partial \eta}\left(\frac{YN}{\mathrm{Sm}} \frac{\partial c_i}{\partial \eta}\right) + f \frac{\partial c_i}{\partial \eta} = 0$$

The following notation is introduced here:

$$u = \tilde{u}/u_\delta; \; f = -\alpha + \int\limits_0^\eta u d\eta, \; g = H/H_\delta, \; N = \rho\mu/p\mathrm{M}_\infty^2, \; Y = r^2/r_w^2 = 1 + kJ(\eta), \; J(\eta) = \int\limits_0^\eta F(h)\, d\eta, \; m = \left(\frac{1}{2} - n\right)\frac{\gamma_\delta - 1}{2\gamma_\delta},$$

$$r^* = r_w(1 + kj(\infty)), \;\; n = 3/4, \qquad k = 2\mathrm{M}_\infty \sqrt{\xi} / \sqrt{\mathrm{Re}}\, p r_w^2$$

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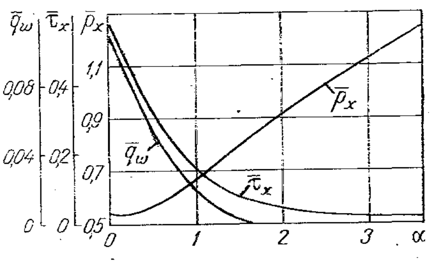

Fig. 1.

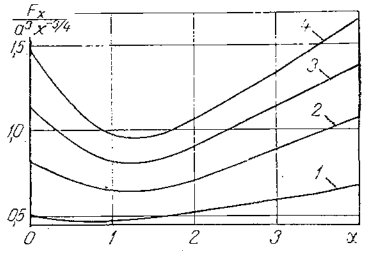

Fig. 2.

Fig. 1. Dependence of $\bar{p}_x$, $\bar{\tau}_x$, and $\bar{q}_w$ on injection rate with $\chi = M_\infty^3/\sqrt{Re_\infty} = 1$, $T_w/T_{0\infty} = \bar{T}_w = 0.25$ ($\bar{p}_x = p_x/a^3x^{-3/4}$, $\bar{\tau}_x = \tau_x/a^3x^{-3/4}$, $\bar{q}_w = q_w/a^3x^{-3/4}$).

Fig. 2. Effect of injection on the total drag of the body with different values of the viscous coupling parameter χ: 1) $\chi = 0.2$; 2) 0.6; 3) 1.0; 4) 1.4.

and is determined from the simultaneous solution of system (2) and the equation

$$k\sqrt{1+kJ(\infty)} = \frac{8}{3\sqrt{c}}\,\frac{\chi}{M_\infty^2 a^2};$$

F(h) is a function in the equation of state $p/\rho = F(h)$. In the case of a single-component gas, $F(h) = h(\gamma - 1)/\gamma$. If the gas in the boundary layer is a mixture, then, assuming that the specific heat of the gases at constant pressure is independent of temperature, we may obtain

$$F(h) = \frac{\gamma_\delta - 1}{\gamma_\delta} h \sum \frac{c_i}{M_i} M_\delta \frac{c_{p\delta}}{c_p}.$$

It is convenient to use the formula of Wickley, Mason, and Saxon [2] to determine the coefficients of molecular transfer of the mixture. For example, for the viscosity coefficient of the binary mixture

$$\mu(T, c_i) = \mu_1(T)\left\{\frac{x_1}{x_1 + G_{12}x_2} + \frac{x_2\dfrac{\mu_2(T)}{\mu_1(T)}}{x_2 + G_{21}x_1}\right\},$$

$$x_1 = \frac{c_1}{c_1 + \dfrac{c_2 M_1}{M_2}},\quad x_2 = \frac{c_2}{c_2 + \dfrac{c_1 M_2}{M_1}},\qquad G_{ij} = \frac{\left[1 + \dfrac{\mu_i(T)}{\mu_j(T)}\left(\dfrac{M_j}{M_i}\right)^{\frac{1}{4}}\right]^2}{2^{\frac{3}{2}}\left(1 + \dfrac{M_i}{M_j}\right)^{\frac{1}{2}}}.$$

Here, the Prandtl number turns out to be dependent on the concentration of the components and their molecular weights.

For N we can obtain

$$N = \gamma_\delta \frac{\mu(T, c_j)}{T} \frac{1}{\sum \dfrac{c_i}{M_i} M_\delta}.$$

In the calculations, results of which are discussed below, a linear dependence of the viscosity of the components on temperature was assumed.

System (2) must be solved with the following boundary conditions:

$$\eta = 0\quad u = 0,\ g = H_w,\ \alpha(c_i - c_i^0) = \frac{N_w}{\mathrm{Sm}}\frac{\partial c_i}{\partial \eta};$$

$$\eta \to \infty\quad u = 1,\ g = 1,\ c_i = c_{i\delta};$$

c_i^0 is the concentration of the i-th component in injected mixture.

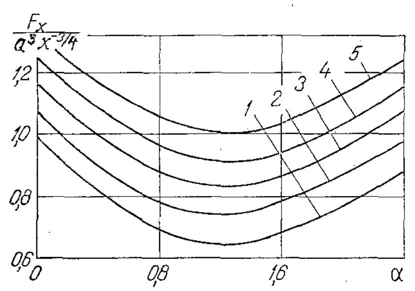

Fig. 3.

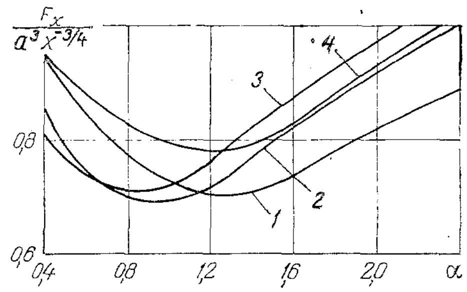

Fig. 4.

Fig. 3. Dependence of total resistance on injection rate at different body-surface temperatures: 1) $\bar{T}_w = 0.2$; 2) 0.4; 3) 0.6; 4) 0.8; 5) 1.0.

Fig. 4. Effect of composition of injected gas on total drag of body at different α: 1) H_2; 2) H_2O; 3) C_3H_6; 4) air.

To determine the mechanical and thermal effects of the flow on the body, we obtain the following relations:

$$p_x = a^3 cn^3 (1 + kJ(\infty))\, x^{-3/4}, \quad \tau_x = dN_w \frac{\partial u}{\partial \eta} x^{-3/4},$$

$$F_x = p_x + \tau_x = a^3 \left(cn^3 (1 + kJ(\infty)) + d' N_w \frac{\partial u}{\partial \eta} \right) x^{-3/4},$$

$$q_w = -dN_w \frac{1}{\Pr} \frac{\partial H}{\partial \eta} x^{-3/4}, \quad d = a^3 d' = a^3 \frac{\chi}{(M_\infty a)^2} \sqrt{1 + kJ(\infty)}\, \frac{3\sqrt{c}}{4},$$

p_x and τ_x are the projections of the pressure and frictional forces onto the symmetry axis of the body; q_w is the heat flux to the surface of the body.

Thus stated, the problem allows us to study the effect of gas injection into the boundary layer on the drag of a thin axisymmetric solid in a hypersonic flow.

The results of this investigation are presented in Figs. 1-4.

The curves in Fig. 1 illustrate the dependence of p_x, τ_x, and q_w on α for a value of the viscous coupling parameter $\chi = 1$. It is apparent that friction and heat flow on the surface of the body decrease and resistance increases with an increase in boundary-layer injection. This increase is connected with the fact that the inviscid flow is forced aside by the injection and the thickness of the "effective" body is increased. Such behavior of frictional forces and wave resistance leads to a nonmonotonic dependence of the total drag of the body on α. This dependence is shown in Fig. 2 for different values of χ. With an increase in χ, the nonmonotonic nature of the dependence of drag on injection rate α becomes more pronounced and the drag minimum shifts to a region of higher α. Figure 2 shows the results for a body surface temperature $T_w = 0.25T_0$. The effect of T_w on total drag at $\chi = 1$ is illustrated in Fig. 3. Drag increases with an increase in temperature. The minimum of F_x is shifted in the direction of higher α. Finally, the curves in Fig. 4 illustrate the dependence of the drag of the body on α in the case of the injection of different gases into the boundary layer: hydrogen, steam, propylene, and air. Here, it is assumed in all cases that the body is in a flow of air. We note the considerable effect of the ratio of molecular weights of the injected gas and the gas of the main flow on the total drag of the body.

The above results provide evidence of the appreciable effect of boundary-layer injection on the drag of a body in a hypersonic flow in the case of substantial viscous coupling. Important here are the temperature of the body surface, the type of gas injected, and the value of the viscous coupling parameter.

NOTATION

$r = r_w(x)$, $r^*(x)$, equations of the surface of the body and effective body in a cylindrical system of coordinates r, x, φ; u, v, ρ, H, p, components of the velocity vector, density, total enthalpy, and pressure, referred to u_∞, ρ_∞, u_∞^2, $\rho_\infty u_\infty^2$, respectively; $\chi = M_\infty^2/\sqrt{Re}$, viscous coupling parameter; M_∞, Re, Mach and Reynolds numbers ($Re = u_\infty \rho_\infty L/\mu_\infty$); μ, coefficient of dynamic viscosity; α, parameter characterizing the injection rate; η, ξ, similarity variables; a, parameter characterizing the thickness of the body; p_x, τ_x, F_x, projection, onto the symmetry axis, of the pressure and friction forces and the total force, referred to $\rho_\infty v_\infty^2$; q_x, heat flux to body surface; c_i, mass concentration of the i-th component of the mixture; M_i, molecular weight of the i-th component; γ, ratio of specific heats at constant pressure and constant volume; c, constant characterizing the pressure on the body surface (c = 1 according to Newton's theory); k, constant for the similarity problem $J(\eta) = \int_0^\eta F(h)d\eta$; $F(h) = p/\rho$; h, specific enthalpy, referred to u_∞^2; T_0, stagnation temperature; c_p, specific heat at constant pressure. Indices: δ, edge of boundary layer; w, surface of body; ∞, hypersonic flow; i, i-th component of mixture.

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DYNAMICS OF GAS COMBUSTION IN A SPHERICAL VESSEL

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Equations are obtained to describe the dynamics of motion of a flame front and a gas in a spherical vessel. The possibility of a transition from combustion to thermal shock is demonstrated.

The combustion of a gas in a spherical vessel with central ignition is the simplest and most frequently theoretically and empirically studied case of combustion with a constant volume. The spherical vessel is often used as a convenient tool to determine the normal rate of propagation of a flame, maximum blast pressure, rate of increase in blast pressure, and certain other parameters of the process which are important for the solution of practical problems of industrial explosion-proofing. Nevertheless, the dynamics of flame propagation in a spherical vessel have yet to be described analytically. The literature [1, 2] contains theoretical descriptions of the dynamics of the pressure increase with an explosion. These descriptions employ differential equations requiring numerical solution on a computer.

As is known, the propagation of a flame in a gas is defined by its normal velocity u and the velocity of the burning gas. The character of the motion of the flame in a closed vessel is complicated by the fact that the velocity of the gas ahead of the flame is variable. However, it can easily be determined on the basis of the condition that the gases formed in front of the flame expand in the direction of the fresh gas and in the direction of the combustion products in a ratio which is proportional to the volumes of fresh gas and combustion products, respectively, at the given moment of time. This condition essentially follows from the condition of equality of the pressures at all points of the vessel during combustion.

A volume udt of fresh gas burns over an infinitesimally short interval of time dt on a unit surface of the flame. If the degree of increase in the volume of the gas with combustion is designated as ε, then the burned gas will occupy the volume εudt and the increment in volume will be $(\varepsilon - 1)$udt. It is this increment that is distributed proportionately as described above. In particular, in the case of a spherical vessel (Fig. 1), the following volume of gas is moved in the direction of the fresh gas

$$\frac{\frac{4}{3}\pi(R^3 - r^3)}{\frac{4}{3}\pi R^3}(\varepsilon - 1)\,udt.$$

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